TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 2 2016

Calculator Assumed
Vectors Chapters 3,4,8

STUDENT'S NAME ______________________________

DATE: **TIME:** 50 minutes **MARKS**: 47

INSTRUCTIONS:
Standard Items: Pens, pencils, ruler, eraser.
Special Items: Three calculators, drawing instruments, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Given $\underset{\sim}{a} = \begin{pmatrix} 24 \\ -7 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 8 \\ 15 \end{pmatrix}$, determine

(a) a unit vector parallel to $\underset{\sim}{b}$ [2]

$|\underset{\sim}{b}| = \sqrt{8^2+15^2} = 17 \quad \therefore \hat{\underset{\sim}{b}} = \frac{1}{17}\begin{pmatrix} 8 \\ 15 \end{pmatrix}$

(b) a vector parallel to $\underset{\sim}{b}$ and four times as long [2]

$4\begin{pmatrix} 8 \\ 15 \end{pmatrix} = \begin{pmatrix} 32 \\ 60 \end{pmatrix}$

(c) the size of the acute angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ [2]

$\cos\theta = \dfrac{\begin{pmatrix} 24 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 15 \end{pmatrix}}{25 \cdot 17} = \dfrac{87}{25.17}$

$\theta = 78.19°$

2. (4 marks)

The unit vector $\hat{\underset{\sim}{\mathbf{u}}} = \begin{pmatrix} a \\ -b \end{pmatrix}$ is perpendicular to $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$. If $a > 0$, determine the values of a and b.

$\begin{pmatrix} a \\ -b \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 0 \Rightarrow 4a - 3b = 0$ — (1)

and $a^2 + b^2 = 1$ — (2)

solving (1) & (2) get $a = \cancel{-0.6}$ or 0.6
$b = \cancel{-0.8}$ or 0.8

$\therefore a = 0.6, \quad b = 0.8$

$\frac{3}{5} \qquad \frac{4}{5}$

3. (5 marks)

Given that $|\underset{\sim}{\mathbf{a}}| = 7$, $|\underset{\sim}{\mathbf{b}}| = 4$ and $\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}} = 11$, determine

(a) $\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{a}}$ [1]

$= 7^2 = 49$

(b) $\underset{\sim}{\mathbf{b}} \cdot \underset{\sim}{\mathbf{b}}$ [1]

$= 4^2 = 16$

(c) $(\underset{\sim}{\mathbf{a}} - \underset{\sim}{\mathbf{b}}) \cdot (\underset{\sim}{\mathbf{a}} - \underset{\sim}{\mathbf{b}})$ [2]

$= |\underset{\sim}{a}|^2 - \underset{\sim}{a} \cdot \underset{\sim}{b} - \underset{\sim}{a} \cdot \underset{\sim}{b} + |\underset{\sim}{b}|^2$

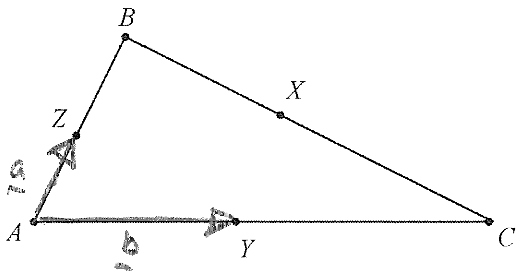
$= 49 - 2(11) + 16 = 43$

(d) $|\underset{\sim}{\mathbf{a}} - \underset{\sim}{\mathbf{b}}|$ exactly [1]

$\sqrt{43}$.

4. (6 marks)

The diagram shows a triangle ABC. Points X, Y and Z are the mid-points of BC, CA and AB respectively.
Vector $\overrightarrow{AZ} = \underset{\sim}{a}$ and vector $\overrightarrow{AY} = \underset{\sim}{b}$



(a) Express in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$

(i) the vector $\overrightarrow{YZ}$ [1]

$\overrightarrow{YZ} = \underset{\sim}{a} - \underset{\sim}{b}$

(ii) the vector $\overrightarrow{CB}$ [1]

$\overrightarrow{CB} = 2\underset{\sim}{a} - 2\underset{\sim}{b} = 2(\underset{\sim}{a} - \underset{\sim}{b})$

(b) Using your answers to part (a), write down <u>two</u> facts about the relationship between the lines YZ and CB. [2]

parallel and CB is twice as long as YZ

(c) Express in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$, the vector $\overrightarrow{AX}$. [2]

$\overrightarrow{AX} = 2\underset{\sim}{b} + (\underset{\sim}{a} - \underset{\sim}{b})$

$= \underset{\sim}{a} + \underset{\sim}{b}$

6. (5 marks)

The position vectors of the points A, B and C are $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively.

Given that $\underset{\sim}{a} = 5\underset{\sim}{c} - 4\underset{\sim}{b}$ determine, in terms of $\underset{\sim}{b}$ and $\underset{\sim}{c}$ only.

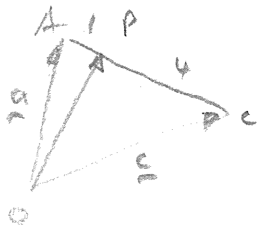
(a) the position vector of M, the mid-point of AB. [2]

$$\vec{OM} = \underset{\sim}{a} + \tfrac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a}) = \tfrac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$$

$$= \tfrac{1}{2}(5\underset{\sim}{c} - 4\underset{\sim}{b} + \underset{\sim}{b})$$

$$= \tfrac{5}{2}\underset{\sim}{c} - \tfrac{3}{2}\underset{\sim}{b}$$

(b) the position vector of the point P which divides AC in the ratio 1 : 4 [3]



$$\vec{OP} = \underset{\sim}{a} + \tfrac{1}{5}(\underset{\sim}{c} - \underset{\sim}{a})$$

$$= \tfrac{4}{5}(5\underset{\sim}{c} - 4\underset{\sim}{b}) + \tfrac{1}{5}\underset{\sim}{c}$$

$$= \tfrac{21}{5}\underset{\sim}{c} - \tfrac{16}{5}\underset{\sim}{b}$$

7. (3 marks)

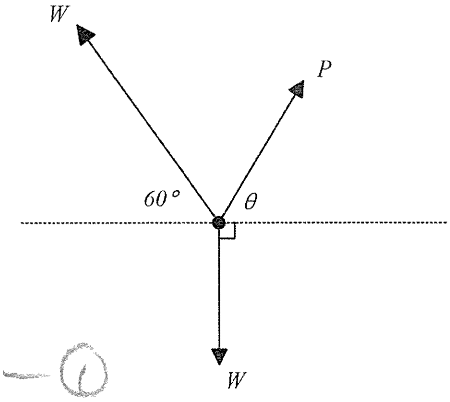
Solve for the vector $\underset{\sim}{a}$ if $2\underset{\sim}{\mathbf{a}} - \begin{pmatrix} 4 \\ -8 \end{pmatrix} = \begin{pmatrix} 17 \\ -36 \end{pmatrix} - 5\underset{\sim}{\mathbf{a}}$

$$7\underset{\sim}{a} = \begin{pmatrix} 21 \\ -44 \end{pmatrix}$$

$$\underset{\sim}{a} = \begin{pmatrix} 3 \\ -\frac{44}{7} \end{pmatrix}$$

8. (5 marks)

The diagram below shows a particle in equilibrium under the forces shown. Determine the **exact** value of $\tan\theta$.



$\rightarrow \quad P\cos\theta = W\cos 60^\circ$

$\uparrow \quad P\sin\theta + W\sin 60^\circ = W$

$\therefore \quad P\sin\theta = W - W\sin 60^\circ \quad — (1)$

$\quad\quad P\cos\theta = W\cos 60 \quad — (2)$

$\div$ (1) by (2) $\quad \tan\theta = \dfrac{\cancel{W}(1-\sin 60^\circ)}{\cancel{W}\cos 60^\circ}$

$= \dfrac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \underline{2-\sqrt{3}}$

OR in component form

for P : $\begin{pmatrix} P\cos\theta \\ P\sin\theta \end{pmatrix}$

for $\nwarrow W$: $\begin{pmatrix} -W\cos 60 \\ W\sin 60 \end{pmatrix}$

for $\downarrow W$: $\begin{pmatrix} 0 \\ -W \end{pmatrix}$

now $\begin{pmatrix} P\cos\theta \\ P\sin\theta \end{pmatrix} + \begin{pmatrix} -W\cos 60 \\ W\sin 60 \end{pmatrix} + \begin{pmatrix} 0 \\ -W \end{pmatrix} = 0$ for equilibrium

⋮

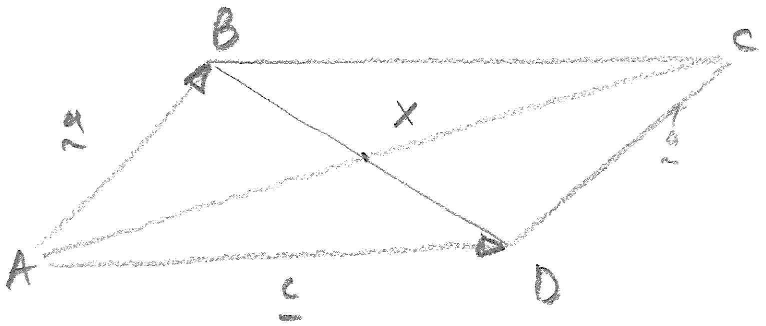
as above

9. (6 marks)

To prove: The diagonals of a parallelogram bisect each other.

$ABCD$ is a parallelogram with $\overrightarrow{AB} = \underset{\sim}{a}$ and $\overrightarrow{AD} = \underset{\sim}{c}$. The diagonals AC and BD meet at X.

If $\overrightarrow{BX} = k\overrightarrow{BD}$ and $\overrightarrow{AX} = t\overrightarrow{AC}$, use the fact that $\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BX}$ to show that $k = t = \frac{1}{2}$



$\overrightarrow{BX} = k\overrightarrow{BD}$

$\overrightarrow{AX} = t\overrightarrow{AC}$

$\overrightarrow{BX} = k(\underset{\sim}{c} - \underset{\sim}{a})$ $\qquad$ $\overrightarrow{AX} = t(\underset{\sim}{a} + \underset{\sim}{c})$

Now $\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BX}$

$t(\underset{\sim}{a} + \underset{\sim}{c}) = \underset{\sim}{a} + k(\underset{\sim}{c} - \underset{\sim}{a})$

$\Rightarrow \quad t = 1 - k$ and $t = k$.

$\therefore \quad t = 1 - t$

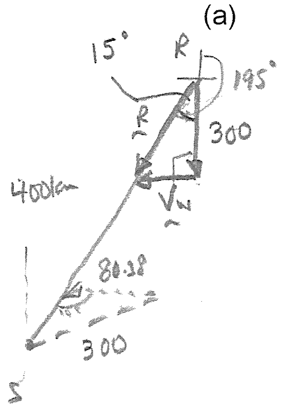
$\therefore \quad 2t = 1$

$t = \frac{1}{2} \quad \therefore k = \frac{1}{2}$

5. (7 marks)

An aircraft, whose speed in still air is 300 kmh^{-1}, flies in a straight line from R to S, a distance of 400 km. The bearing of S from R is 195°. There is a wind blowing from the east. Given that the pilot needs to set a course due south, calculate

(a) the speed of the wind [2] 3



$\tan 15^\circ = \frac{|V_w|}{300}$ $\therefore |V_w| = \underline{80.38 \text{ kmh}^{-1}}$

(b) the time, in minutes, of the flight [2]

$|R| = \frac{300}{\cos 15^\circ}$

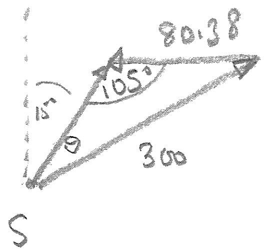
$= 310.58$

$t = \frac{400}{|R|} = \frac{400}{310.58} = 1.29 \text{ hrs}$

$= \underline{77.3 \text{ mins}}$

If the speed and direction of the wind are unchanged

(c) What course does the pilot need to set on the return flight from S to R? [3]



$\frac{300}{\sin 105^\circ} = \frac{80.38}{\sin \theta}$

$\theta = 14.999^\circ \approx 15^\circ$

$\text{bearing} = \underline{030^\circ}$